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OPTIMAL CONTROL AND COST-EFFECTIVENESS ANALYSIS OF INFECTIOUS CORYZA DISEASE EPIDEMIC MODEL

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ABSTRACT

This paper concentrates on the dynamics, control measure and cost-effectiveness analysis of Infectious Coryzadisease (ICD). We examined the boundedness of ICD model in the R_+^4. Different controls types used to determine the optimal level of strategy that decreases the spreads of the diseases as well as the cost of implementation of the control. Simulation of different variables of the model was performed to justify the analytical results and we have used Pontryagin's maximum principle to derive necessary conditions for the optimal control of the disease. The cost-effectiveness analysis results show that an optimal effort on vaccination and treatment are the most cost-effective strategies to combat the epidemic of Infectious Coryza with limited resources. Therefore, ICDcan is controlled if the farmers will usevaccinationand treatment control properly.

KEYWORDS:

Infectious Coryza diseases**;**
Optimal control;
Cost-effectiveness

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1. INTRODUCTION

Infectious Coryza (IC) is an acute respiratory disease of chickens caused by the bacterium, Avibacteriumparagallinarumpreviouslyknown as Haemophilusparagallinarum[1]. This bacterium is from the genus Haemophilus, which is a member of the family Pasteurellaceae[1]andaparagallinarum. It usually affects the upper respiratory tract of

chickens [1]. The species affected most are chickens, pheasants, and guinea fowl[2]. This study focuses on the dynamics of ICD in chicken because are the natural host for Avibacteriumparagallinarum. The disease is characterized by nasal discharge, facial swelling, sneezing, diarrhoea and anorexia [3], which lead to a drop in egg production, decreased feed and water consumption [4]. In different areas worldwide the diseases have been reported such as Argentina, Australia, Germany, India, Indonesia, Japan, Malaysia, Mexico, Morocco, Pakistan, Taiwan, Thailand and Uganda [5].

Several studies were conducted to assess the distribution and prevalence of ICD in Tanzania. It was found that in Arusha ninety-nine of 2216 (4.4%) chickens were diagnosed as having died of bacterial-associated infections and their distribution is as follows: Omphalitis, 26 of 99(26.3%), Colibacillosis, 21 of 99(21.3%), salmonellas is, 18 of 99(18.1%), pasteurellosis, 18 of 99(18.1%) and infectious coryza 16 of 99(16.2%).

Despite the availability of the infectious coryza vaccine, the disease is still endemic in a different area of the world including developed nations[3]. Even though the government of Tanzania through different means has tried to emphasize on the vaccination and treatment of infectious coryza disease [6], the disease has continued to cause economic depression for both government and individuals who invest in chicken. The susceptible chicken population may get the infection through direct contact, airborne droplets, and through contaminated drinking water. The transmission among bird to bird normally occur through contact with contaminated drinking water [1].

The incubation period ranges from 1-3 days, and the disease usually ranges from 2-3 weeks.

One of the methods to decrease the dynamic of the disease in chicken is to keep the house clean, change the poultry liter, be careful add birds, use quality feed and proper spacing. Another remarkable measure in outbreak cases is the segregation of birds by age [5]. Treatment can improve the spread of the diseases but recur when medication is discontinued. Therefore, water medication is recommended as a form of early treatment.

Due to this impact, this study aims at developing a mathematical model as an alternative approach for controlling the spread of the disease to help farmers fight against the outbreak of infectious coryza disease.

2. MATERIALS AND METHODS

This section, we formulate a deterministic, compartmental model to describe the dynamics of infectious coryza disease among the free-range indigenous chickens. We assumed that chickens are homogeneously mixing. It further, assumed that chickens can be infected through direct contact with an infected chicken, and can recover through treatment however; they may be infected again if they come in contact with the infected chickens. The model is composed of four compartments namely susceptible chickens (s_c), exposed chickens (E_c), infectious chicken (I_c), and contaminated environment (W). The susceptible class increases through recruitment rate π and by recovery through treatment rate α . β is the transmission rate from susceptible chickens to exposed chickens, γ is the progress rate from exposed to infectious. Chicken may also get the infection through the contaminated environment and progress to infectious at the rate λ or can progress to exposed chicken at a rate θ while the remaining fraction of the populations moves to susceptible class at rate δ. It is assumed that chickens may die naturally at a rate μ, while d describes the environmental intrinsic growth rate, k is the environmental carrying capacity and ε represents the environmental cleanliness rate. The model parameters are therefore summarized in Table 1.

Table1. Parameters' Descriptions

Parameters	Description
α	Treatment rate of infectious chickens
β	Transmission rate from susceptible chicken to exposed chickens.
d	Environmental intrinsic growth rate.
k	The rate of dirty.
λ	Rate of transmission from Contaminated environment to infectious chickens.
μ	The natural death rate of chickens
θ	Rate of transmission from contaminated environment to exposed chickens.
γ	The progressive rate from exposure to infectious chickens
π	Chickens growth rate/recruitment rate
δ	Progression of none infected chicken to susceptible class

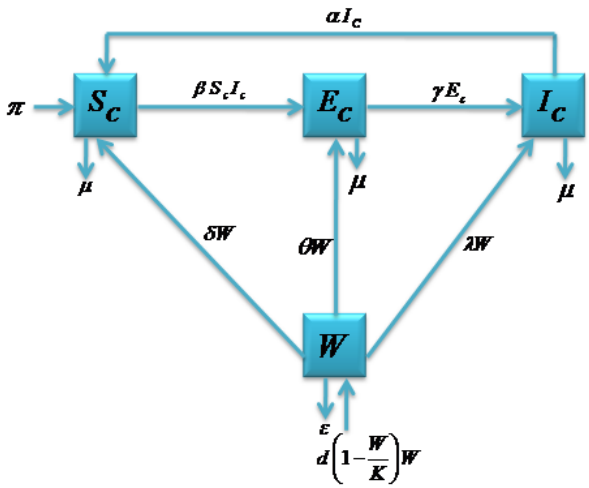

Figure 1: Flow diagram of the model

Using Figure 1, the following set of differential equations are generated:

$$\left.\begin{aligned}
\frac{dS_c}{dt} &= \pi + \alpha I_c + \delta W - \beta S_c I_c - \mu S_c \\
\frac{dE_c}{dt} &= \beta S_c I_c + \theta W - (\mu + \gamma) E_c \\
\frac{dI_c}{dt} &= \lambda W + \gamma E_c - (\mu + \alpha) I_c \\
\frac{dW}{dt} &= d\left(1 - \frac{W}{k}\right) W - (\delta + \varepsilon + \theta + \lambda) W
\end{aligned}\right\} \quad (1)$$

With initial $S_c > 0, E_c \geq 0, I_c \geq 0, W \geq 0$

2.1. Boundedness

The model equation (1) used to test if all state variables and parameters are non-negative for all $t \geq 0$. It is used to show that all state variables remain positive for all positive initial values.

The boundedness of the model (1) evaluated through lemma 1.

Lemma 1.1

All the solution of the system (1) which starts in R_+^4 is uniformly bounded.

Proof:

Let

$$k(t) = S_c(t) + E_c(t) + I_c(t) \tag{2}$$

Differentiating equation (2) we get:

$$\frac{dk}{dt} = \frac{dS_c}{dt} + \frac{dE_c}{dt} + \frac{dI_c}{dt} = \pi - \mu(S_c + E_c + I_c)$$

$$\frac{dk}{dt} = \pi - \mu k$$

(3)

Solving (3) we get

$$K(t) = \frac{\pi + d(1 - \frac{w}{k}) - \varepsilon w}{\mu} + Ce^{-\mu t} \tag{4}$$

Then

$$0 \le k(t) \le \frac{\pi + d(1 - \frac{w}{k}) - \varepsilon w}{\mu} + Ce^{-\mu t} + (S_c(0) + E_c(0) + I_c(0) + W(0))e^{-\mu t} \tag{5}$$

Consequently, as $t \to \infty$, $k(t) < \infty$ we have

$$0 \le k(t) \le \frac{\pi + d(1 - \frac{w}{k}) - \varepsilon w}{\mu} \tag{6}$$

For any $\varepsilon > 0$ is bounded. This implies that all solutions of the system (1) are uniformly bounded in the interior of R_+^4.

2.2 Existence and uniqueness of the solution of the model

In this subsection, we formulate the conditions for the existence and uniqueness of a solution for the model (1). Thus the model is presented as:

$$\left.\begin{aligned}
\frac{dS_c}{dt} &= \pi + \alpha I_c + \delta W - \beta S_c I_c - \mu S_c, \quad S_c(t_0) = S_0 \\
\frac{dE_c}{dt} &= \beta S_c I_c + \theta W - (\mu + \gamma) E_c, \quad E_c(t_0) = E_0 \\
\frac{dI_c}{dt} &= \lambda W + \gamma E_c - (\mu + \alpha) I_c, \quad I_c(t_0) = I_0 \\
\frac{dW}{dt} &= d\left(1 - \frac{W}{k}\right) W - (\delta + \varepsilon + \theta + \lambda) W, W(t_0) = W_0
\end{aligned}\right\} \tag{7}$$

Theorem 2.1

Let $$D=\left|\left(S_c,E_c.I_c,W\right)\right|\left|S_c-S_0\right|\le b,\left|E_c-E_0\right|\le b,\left|I_c-I_0\right|\le b,\left|W-W_0\right|\le b,b\in R^+. \quad (8)$$

and suppose that $f(t,x)$ satisfies the Lipchitz condition

$$\left\|\left(t,x_1\right)-f\left(t,x_2\right)\right\|\le M\left\|.\right\| \quad (9)$$

which is defined as

$\frac{\left|f(x)-f(x+h)\right|}{h}<M$, then, $\left|f'(x)\right|<M\in R^+$ has a unique solution.

Proof

From equation (7) we have:

$$\left.\begin{aligned}
\frac{dS_c}{dt} &= f_1(S_c,E_c.I_c,W)= \pi+\alpha I_c+\delta W-\beta S_c I_c-\mu S_c\\
\frac{dE_c}{dt} &= f_2(S_c,E_c.I_c,W)=\beta S_c I_c+\theta W-\left(\mu+\gamma\right)E_c\\
\frac{dI_c}{dt} &= f_3(S_c,E_c.I_c,W)=\lambda W+\gamma E_c\ -\left(\mu+\alpha\right)I_c\\
\frac{dW}{dt} &= f_4(S_c,E_c.I_c,W)= d\left(1-\frac{W}{k}\right)W-\left(\delta+\varepsilon+\theta+\lambda\right)W
\end{aligned}\right\} \quad (10)$$

Then, need to show that equation (10) evaluated as

$$\left|\frac{\partial f_i}{\partial x_i}\right|,i,\ j=1,2,3$$

are continuous and bounded. Then it shows that the system (7) has a unique solution.

2.3 Application of optimal control to the ICD model

The time-dependent control is introduced in the model (1) for the purpose of studying and analyzing different strategies that can be used in controlling the ICD epidemic in chicken. The following assumptions are used as a guideline in formulating a system of differential equations as an optimal control problem. It is assumed that infected chickens may be controlled through treatment and denoted as $u_3(t)$. It is also assumed that susceptible

populations are protected through vaccination $u_1(t)$. Not only but that also the disease can be controlled through environmental cleanliness which is denoted by $u_2(t)$. Further, it is assumed that a fraction of susceptible population being infectious is $(1-u_1(t))$, while the remaining population turns to a class of susceptible. The incorporated control time is bounded by $t \in [0,T]$, where T represents the final time of the intervention program. The vaccination control will be evaluated at its optimal level when $u_1 = 1$ and at the minimum level when $u_1 = 0$. The control associated with chickens environmental cleanliness attains its maximum level whenever $u_2 = 1$ and the optimal level of treatment is achieved when $u_3 = 1$ Otherwise, it is assumed that intervention is at a low or intermediate level. Hence, incorporating these assumptions in the model (1), we generate the following model equations:

$$\left.\begin{aligned}
\frac{dS_c}{dt} &= \pi + (\alpha + u_3) I_c + \delta W - (1-u_1)\beta S_c I_c - \mu S_c \\
\frac{dE_c}{dt} &= (1-u_1)\beta S_c I_c + \theta W - (\mu + \gamma) E_c \\
\frac{dI_c}{dt} &= \lambda W + \gamma E_c - (\mu + u_3 + \alpha) I_c \\
\frac{dW}{dt} &= d\left(1 - \frac{W}{k}\right) W - (\delta + u_2 + \varepsilon + \theta + \lambda) W
\end{aligned}\right\} \qquad (11)$$

It is assumed that the control strategies that are chicken vaccination, treatment of infected chickens and chicken environmental sanitation has maximum limitations in a given period of time. The limitations are evaluated under a Lebesgue measurable control variable presented as

$$u = \{u = (u_1, u_2, u_3), 0 \le u_i \le u_{i\max}, i = 1,2,3\}.$$

This leads to the minimization of the number of the infected chicken population while minimizing the associated cost of interventions u_1, u_2 and u_3 in a specified period of time. Thus, the optimal control problem is set to minimize the objective functional[8,9] as

$$J(u)=\int_0^T\left(A_1I_c+A_2W+\frac{B_1}{2}u_1^2+\frac{B_2}{2}u_2^2+\frac{B_3}{2}u_3^2\right)dt \tag{12}$$

where $A_i>0$ represents the weight of control of the infected chickens and their environment, $B_i>0$ represents the relative weight of control cost and benefit of the control while $\frac{B_1u_1^2}{2}$ is the minimization cost of vaccination control, $\frac{B_2u_2^2}{2}$ is the minimization cost of environment cleanliness and $\frac{B_3u_3^2}{2}$ is the minimization cost of treatment control.

The aim here is to find the pair of optimal control $u^*=\left(u_1^*,u_2^*,u_3^*\right)$ such that:

$$J\left(u^*\right)=\min_U J\left(u_1,u_2,u_3\right) \tag{13}$$

The basic setup of the optimal control problem is to check the existence and uniqueness of the optimal controls and to characterize them[8,- 13].

2.4 The existence of optimal controls

Theorem 2.2

Given $J(u)$ subject to the system (12) with $\left(S_c(0),E_c(0),I_c(0),W(0)\right)\geq(0,0,0,0)$ then, there exists an optimal control u^* and corresponding $\left(S_c^*,E_c^*,I_c^*,W^*\right)$, that minimizes $J(u)$ over U. The proof for a particular theory is based on assumptions presented by [14,15], that:

The set of controls and corresponding state variables is nonempty.

The measurable control set is convex and closed.

Each right-hand side of the state system is continuous and bounded above by a sum of the bounded control and the state, it can be written as a linear function of u with coefficients depending on time and the state[12].

Suppose there exist constants $q_1,q_2>0$, and $\mathrm{B}>1$ such that the integrant of the objective functional satisfies

$$J\geq q_1\left(|u_1|^2+|u_2|^2+|u_3|^2\right)^{\frac{\mathrm{B}}{2}}-q_2.$$

Proof

If U is a nonempty set of measurable functions on $0 \le T$ and let its values be in real numbers . The system (11) has bounded coefficients and therefore any solutions of the system are bounded on $[0,T]$. Hence, the corresponding solutions for the system (11) exist.

It suffices to write $U = U_1 \times U_2 \times U_3$. So that $U = U_1 \times U_2 \times U_3$ is bounded and convex $\forall t \in [0,T]$.

Each right-hand side of the system (11) by definition is continuous and all variables S_c, E_c, I_c, W and u are uniformly bounded on $[0,T]$. To check for the boundedness of the system (11), we apply the concept of a super solution and system (11) is now expressed as:

$$\left.\begin{aligned} \frac{dS_c}{dt} &= \pi + (\alpha + u_3) I_c + \delta W \\ \frac{dE_c}{dt} &= \beta S_c I_c + \theta W \\ \frac{dI_c}{dt} &= \lambda W + \gamma E_c \\ \frac{dW}{dt} &= dW \end{aligned}\right\} \qquad (14)$$

are bounded on the finite interval. The system can be written as:

$$\begin{bmatrix} S_c^{'} \\ E_c^{'} \\ I_c^{'} \\ W^{'} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \alpha + u_3 & \delta \\ \beta \hat{I}_c & 0 & \beta \hat{S}_c & \theta \\ 0 & \gamma & 0 & \lambda \\ 0 & 0 & 0 & d \end{bmatrix} \begin{bmatrix} \hat{S}_c \\ \hat{E}_c \\ \hat{I}_c \\ \hat{W} \end{bmatrix} + \begin{bmatrix} \pi \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (15)$$

As it can be seen the system is nonlinear infinite time with bounded coefficients and the super solutions of

$\hat{S}_c, \hat{E}_c, \hat{I}_c, \hat{W}$ are uniformly bounded. Since the solution to each state equation is bounded, then:

$$\left| f(t,\ S_c,\ E_c,\ I_c,\ W,\ u \right| \le \left\| \begin{bmatrix} 0 & 0 & \alpha & \delta \\ \beta \hat{I}_c & 0 & \beta \hat{S}_c & \theta \\ 0 & \gamma & 0 & \lambda \\ 0 & 0 & 0 & d \end{bmatrix} \begin{bmatrix} \hat{S}_c \\ \hat{E}_c \\ \hat{I}_c \\ \hat{W} \end{bmatrix} + \begin{bmatrix} \pi \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\| + I_c \left\| \begin{bmatrix} u_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\| \qquad (16)$$

$\leq k_1|X|+I_c|u_3|+k_2$

where X=S_c, E_c, I_c, W, k_1 depends on the coefficient of the system. Thus, the assumption holds.

Let f be the integrant of the objective functional (12)which is articulated as

$$f = A_1 I_c + A_2 W + \frac{B_1}{2}u_1^2 + \frac{B_2}{2}u_2^2 + \frac{B_3}{2}u_3^2$$

Then, f can be expressed inequality as

$$f \geq \frac{q_1}{2}\left(u_1^2 + u_2^2 + u_3^2\right) - q_2$$

Where $q_1 = \min\{B_1, B_2, B_3\}$, $B = 2$ and $q_2 > 0$. Thus, the assumption holds. Therefore optimal control u exists.

2.6 Characterization of the optimal controls

The study applies the optimal controls that rely on Pontryagin's maximum principle as presented by [14] and applied by many other authors. To use this theory, we convert the optimal control problem (11) and objective functional (12) into a problem of minimizing point-wise a Hamiltonian (H), with respect to $u(t)$. The Hamiltonian equation is formed as:

$$\begin{aligned} H = {} & A_2 W + A_1 I_c + \frac{1}{2}B_1 u_1^2 + \frac{1}{2}B_2 u_2^2 + \frac{1}{2}B_3 u_3^2 + L_1\begin{Bmatrix}\pi + (\alpha + u_3)I_c + \delta W \\ -(1-u1)\beta S_c I_c - \mu S_c\end{Bmatrix} + \\ & L_2\left\{(1-u_1)\beta S_c I_c + \theta W - (\mu+\gamma)E_c\right\} + L_3\begin{Bmatrix}\lambda W + \gamma E_c - \\ (\mu + u_3 + \alpha)I_c\end{Bmatrix} + \\ & L_4\left\{d\left(1-\frac{W}{k}\right)W - (\delta + u_2 + \varepsilon + \theta + \lambda)W\right\} \end{aligned} \quad (17)$$

The optimal control and its corresponding state variable are on an optimal couple that satisfies the adjoint equation, optimality and transversality conditions with the adjoint vector

$$L = (L_1, L_1, L_3, L_4).$$

That is

$$\frac{dX}{dt} = \frac{\partial H(X,u,L)}{\partial L}$$
$$\frac{dL}{dt} = -\frac{\partial H(X,u,L)}{\partial X}$$
$$\frac{\partial H(X,u,L)}{\partial u} = 0 \qquad (18)$$

Theorem 2.3

Given an optimal control u^* and solution of the corresponding state X^*, then, there exists an adjoint vector L which satisfies the following:

$$\left.\begin{aligned}
\frac{dL_1}{dt} &= -L_1\left(-(1-u_1)\beta I_c - \mu\right) - L_2(1-u_1)\beta I_c \\
\frac{dL_2}{dt} &= -L_2(-\mu-\gamma) - L_3\gamma \\
\frac{dL_3}{dt} &= -A_1 - L_1\left(\alpha + u_3 - (1-u_1)\beta S_c\right) - L_2(1-u_1)\beta S_c - L_3(-\mu-u_3-\alpha) \\
\frac{dL_4}{dt} &= -A_2 - L_1\delta - L_2\theta - L_3\lambda - L_4\left(-\frac{dW}{k} + d\left(1-\frac{W}{k}\right) - \delta - u_2 - \varepsilon - \theta - \lambda\right)
\end{aligned}\right\} \qquad (19)$$

Transversality condition $L_i(T) = 0$, then optimality condition satisfies given by

$$u_1^* = \max\left\{0, \min\left\{1, \frac{\beta S_c I_c (L_2 - L_1)}{B_1}\right\}\right\}$$
$$u_2^* = \max\left\{0, \min\left\{1, \frac{WL_4}{B_2}\right\}\right\}$$
$$u_3^* = \max\left\{0, \min\left\{1, \frac{I_c (L_3 - L_1)}{B_3}\right\}\right\} \qquad (20)$$

Proof:

The adjoint equations (19) obtained by differentiating Hamiltonian function (17) with respect to X=S_c, E_c, I_c, W and is defined as

$$\frac{dL}{dt} = -\frac{\partial H(X,u,L)}{\partial X}.$$

Let the final states $S_c(T)$, $E_c(T)$, $I_c(T)$, $W(T)$ be free and this will lead transversality condition $L(T) = 0$. For the case of optimality condition, we use the differentiate the Hamiltonian function

With respect to u_i as:

$$\frac{\partial H(X,u,L)}{\partial u}=0$$

and we obtain

$$\left.\begin{aligned}
&\frac{\partial H}{\partial u_1}=\beta S_c I_c L_1-\beta S_c I_c L_2+u_1 B_1 \Rightarrow u_1^*=\frac{\beta S_c I_c (L_1-L_2)}{B_1}\\
&\frac{\partial H}{\partial u_1}=-WL_4+u_2 B_2 \Rightarrow u_2^*=\frac{WL_4}{B_2}\\
&\frac{\partial H}{\partial u_1}=I_c(L_1-L_3)+u_3 B_3 \Rightarrow u_1^*=\frac{I_c(L_3-L_1)}{B_3}
\end{aligned}\right\} \quad (21)$$

We also check the concavity conditions of the objective function through

$$\frac{\partial^2 H}{\partial u^2}>0,$$

the second derivative of a Hamiltonian function is

$$\frac{\partial^2 H}{\partial u^2}=\begin{pmatrix} B_1 & 0 & 0\\ 0 & B_2 & 0\\ 0 & 0 & B_3 \end{pmatrix} \quad (22)$$

Since the weight of the control $B_i>0$, then the matrix (22) is positive and therefore the optimal control is minimized.

3.0 RESULTS AND DISCUSSION

Solving model (11) and its corresponding adjoint equations (19) with optimality condition that is manipulated as u^* in terms of costate variables form the boundary value problem which can be solved by many numerical methods such as Runge-Kutta, adaptive scheme, shooting method to list but few. In this paper, the Runge-Kutta method using MATLAB program is used to solve four-state equations and four adjoint equations. The optimal control problem (11) solved by a forward method with the initial guess values and then transversality conditions are used as initial values. For the case of adjoint equations (19), the backward approach is used in solving the current iterations solution of the state system. The controls are, therefore, updated by using a convex combination of the previous controls. The convergence criteria are evaluated with negligibly small error evaluated with

$$\frac{\left\|\vec{u} - old\vec{u}\right\|}{\left\|\vec{u}\right\|} \leq \sigma$$

Where σ is acceptance tolerance.

3.1 Numerical Simulations

This section presents, numerical simulations of the system (11) by analyzing different control strategies to combat the ICD in chickens. The optimal control strategies are formulated and presented as follows:

Strategy A: control with chicken vaccination and environmental cleanliness (u_1, u_2)

The results show the positive impact of reducing the spread of disease to the system with chicken vaccination and environmental cleanliness as shown in Figure 2(a-b). The susceptible chicken tends to increase whenever vaccination and environmental control strategy are applied and also it is observed that exposed chickens to ICD decrease as the impact of control is as shown in Figure 2(c). The control profile suggests that the control u_1 is at the upper bound for the approximation of six days before dropping gradually to the lower bound while control u_2 is at the upper bound for about one month before dropping to the lower bound (see Figure 2(d)

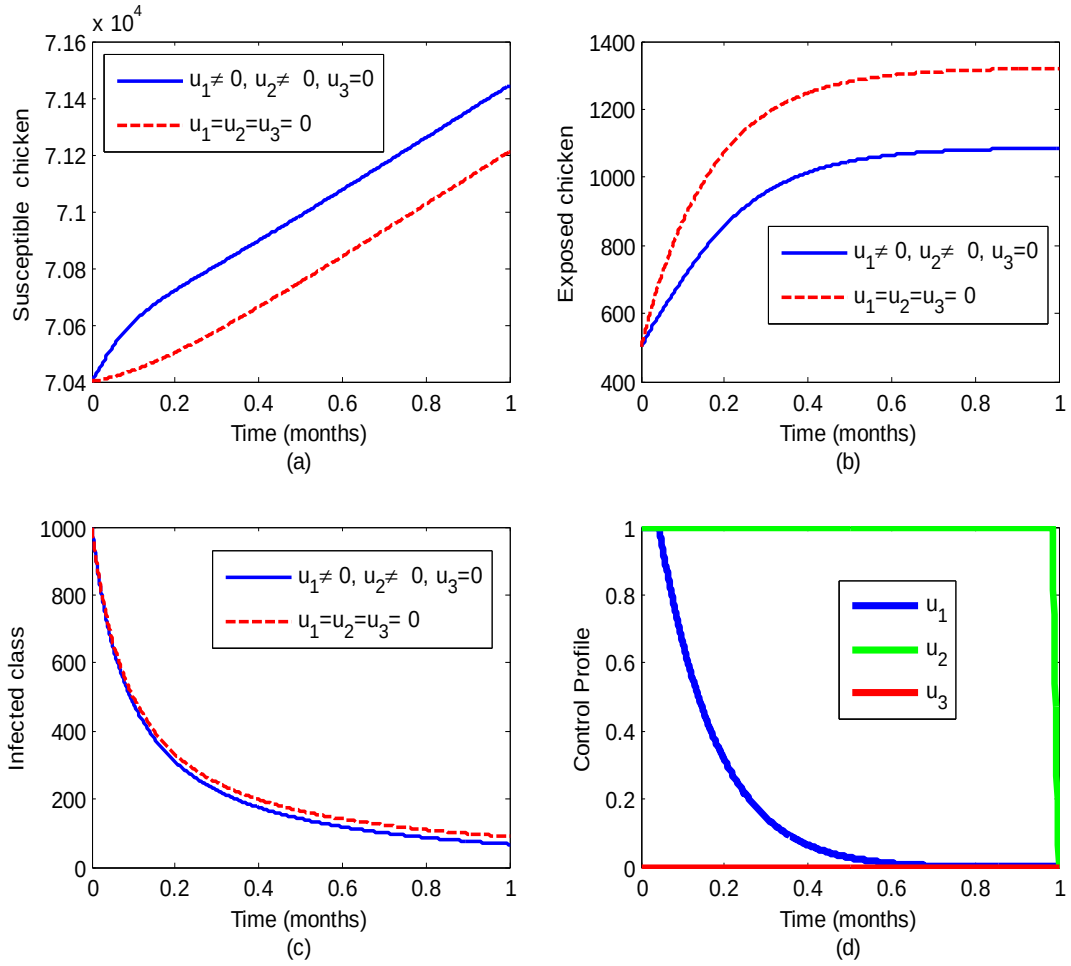

Figure 2: Application of Chicken Vaccination and environmental cleanliness to the chicken population; with parameter values in Table 2.

Strategy B: Control with environmental cleanliness and infected chicken treatment (u_2, u_3)

When u_2 and u_3 are applied to the system and u_1 maintained at zero, the results show that susceptible chickens increase and infected chickens decrease as shown in Figure3 (a) and Figure 3 (c). It is also observed that when control is applied to the exposed chickens, it reduces the number of chickens to be affected by ICD. The control profile suggests that the control u_2 is at the upper bound for one month before it drops to the lower bound. While the control u_3 to be at the upper bound for just twenty-four days before dropping to the lower bound. This result shows that the treatment control strategy is not effective when applied to chickens before susceptible among them are vaccinated.

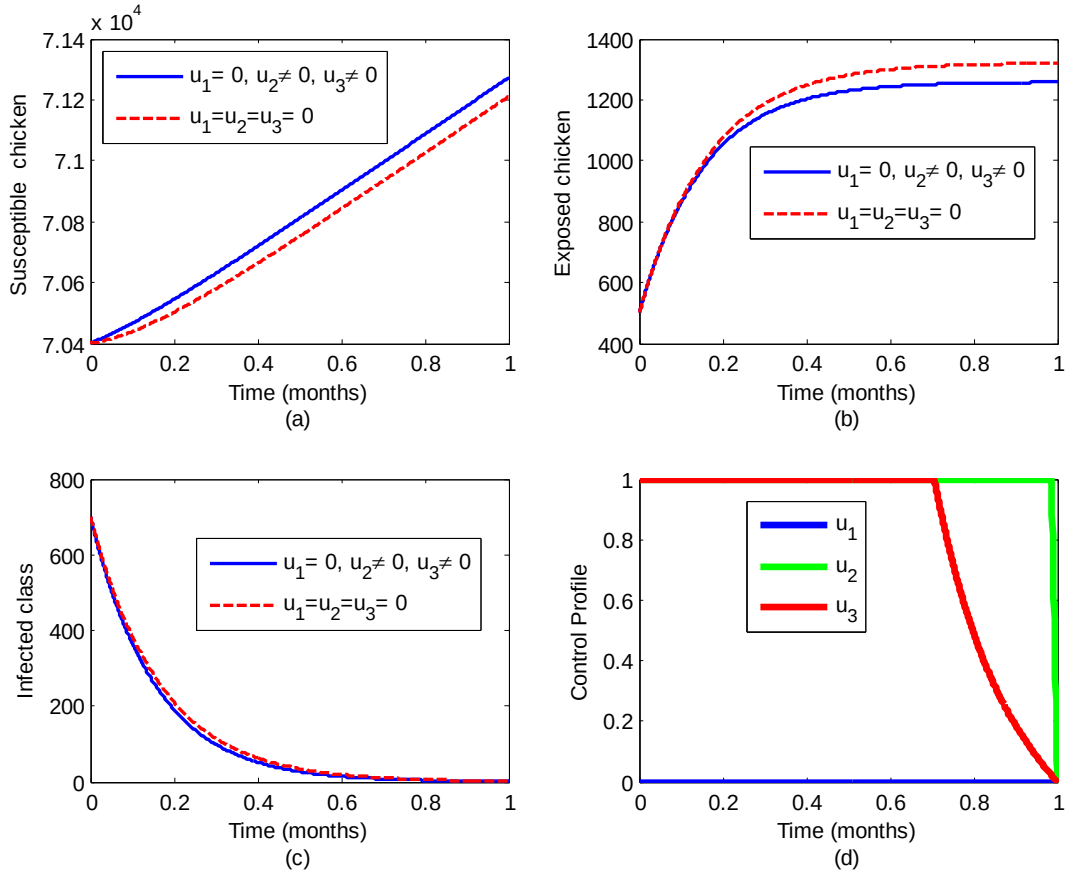

Figure 3: Variation of the chicken population as the impact of environmental cleanliness and

infected chicken treatment (u_2, u_3), with parameter values in Table 2.

Strategy C: Control with chicken vaccination and treatment (u_1, u_3)

When u_1 and u_3 are applied to the system and u_2 is set to zero the results show that susceptible chickens increase and infected chicken decrease as shown in Figure4(c). It is also observed that without control, exposed chickens increase and when control is applied, reduce the number of chicken affected by ICD. The control profile shows the control u_1 is at the upper bound for three days before dropping to the lower bound while the control u_3 to be at the upper bound for just twenty-four days before dropping gradually to the lower bound. The result shows that the treatment control strategy is more effective when applied after vaccination of the susceptible chickens.

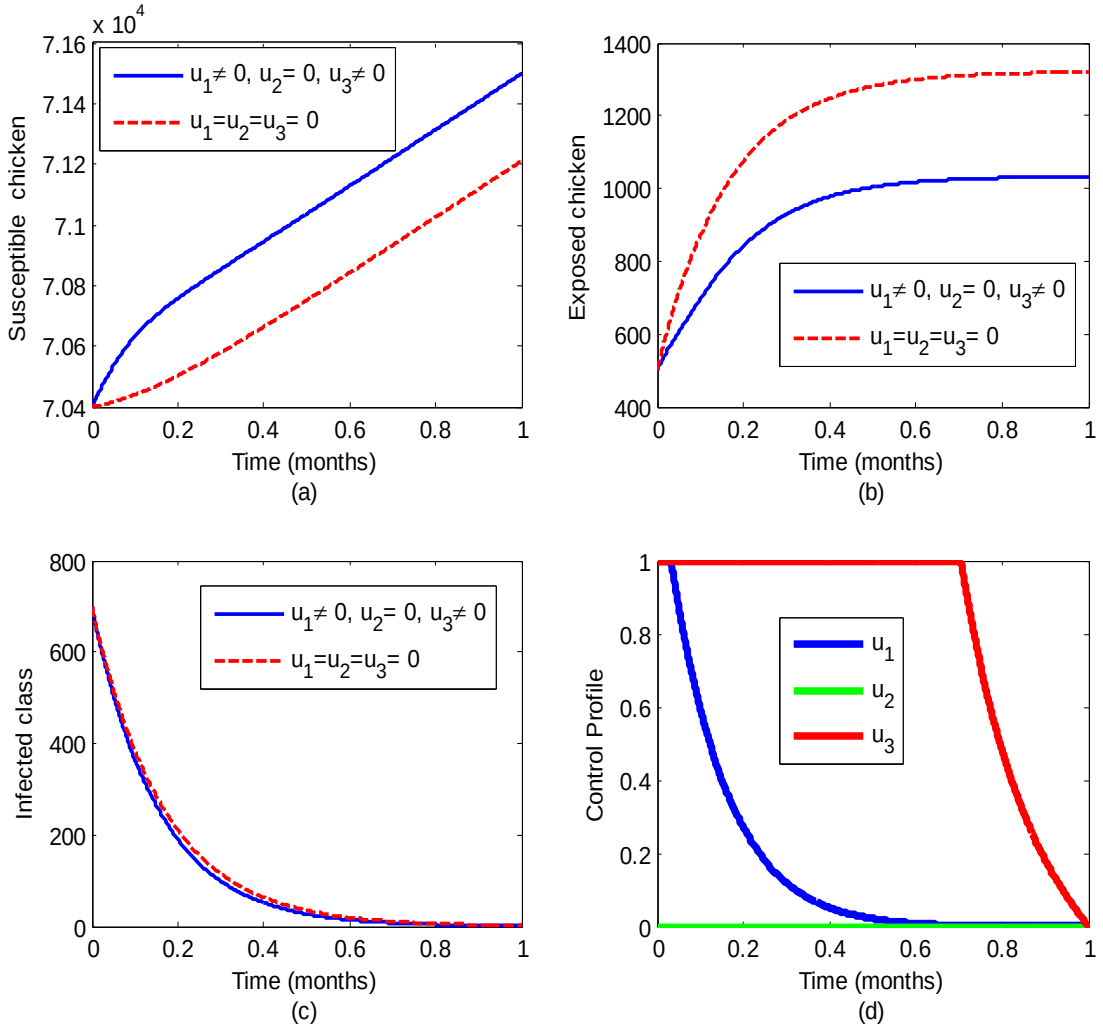

Figure 4: Variation of the chicken population as the impact of chicken vaccination and treatment (u_1, u_3), with parameter values in Table 2

Strategy D: control with chicken vaccination, environmental cleanliness and treatment (u_1, u_2, u_3)

The results show the positive impact of reducing the spread of disease to the system with chicken vaccination, environmental dirtiness and treatment as shown in Figure 5, Susceptible chickens increase when the control is applied and also when control is applied the number of exposed chickens to ICD decreases. However, infected chickens decrease to zero when control is applied as shown in Figure (c). The control profile shows that the control u_1 is at the upper bound for just three days before dropping gradually to the lower

bound as well as control u_2 to be at the upper bound for one month and u_3 for twenty-four days before dropping to the lower bound.

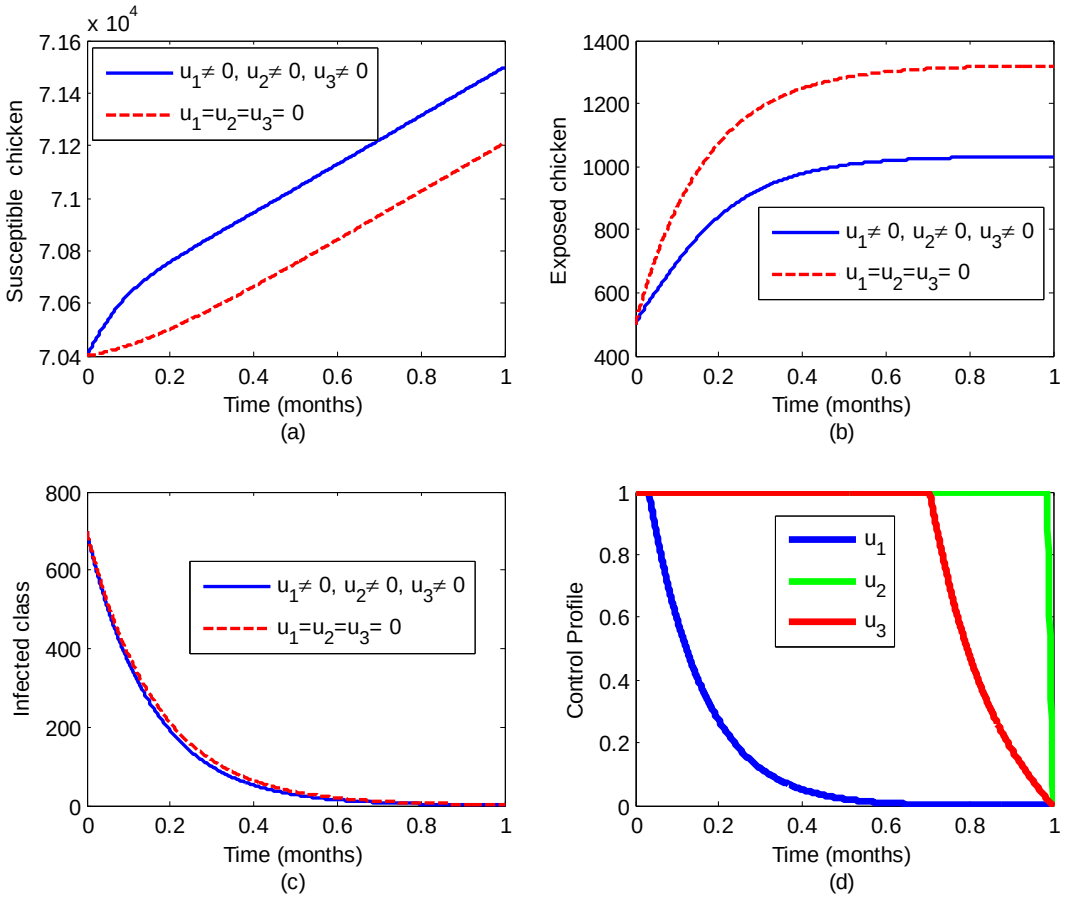

Figure 5: Variation of the chicken population as the impact of chicken vaccination, environmental cleanliness and infected treatment (u_1, u_2, u_3), with parameter values in Table.

3.2 Cost-effective analysis

The cost-effectiveness is assessed through incremental cost-effectiveness ratio (ICER) that compares the costs and health outcomes of the two competing intervention strategies. Each intervention is compared with the next less effective alternative [12]. The averted chickens are computed by finding the difference between the total number of chickens without control and the total number of chickens with control. In making a decision on which intervention to implement in limited resources, the economic evaluation of ICD is carried out to determine and propose the most cost-effective strategy(15). The total control cost is evaluated as:

$$C(u) = \underset{u_1,u_2,u_3}{Min} \int_0^3 \left(\frac{1}{2} B_1 u_1^{\,2} + \frac{1}{2} B_2 u_2^{\,2} + \frac{1}{2} B_3 u_3^{\,2} \right) dt$$

The total control costs $B_1 u_1^2$, $B_2 u_2^2$ and $B_3 u_3^2$ were B_i for $i = 1, 2, 3$ is relative cost weight for each control measure.

Table 2: control strategies with the cost

Strategies	Averted species	Control costs $(\$)$	Total costs J $(\$)$
Strategy A($u_1 \neq 0, u_2 \neq 0, u_3 = 0$)	234	0.2753	167.2676
Strategy B($u_1 \neq 0, u_3 \neq 0, u_2 = 0$)	289	0.2159	173.4895
Strategy C($u_2 \neq 0, u_3 \neq 0, u_1 = 0$)	63	0.4397	163.0295
Strategy D($u_1 \neq 0, u_2 \neq 0, u_3 \neq 0$)	288	0.4634	163.0264

The numerical outputs for the control strategies in Table 2 are ranked in increasing order of effectiveness in the form of infection averted as presented in Table 3.

Table 3: control strategies in order of increasing averted

Strategies	Total infections averted	Control costs $(\$)$	Total costs J $(\$)$
Strategy C	63	0.4397	163.0295
Strategy A	234	0.2753	167.2676
Strategy D	288	0.4634	163.0295
Strategy B	289	0.2159	173.4895

We calculate and compare the cost-effectiveness ratio (ICER) for strategy C and A as shown in Table 4.

Table 4: Total infections averted, total cost and ICER.

Strategies	Total infections averted	Total costs $(\$)$	ICER
Strategy C	63	0.4397	0.0069793651
Strategy A	234	0.2753	-0.000961403

The ICER is calculated as follows;

$$ICER = \frac{Difference\ in\ costs\ in\ strategies\ i\ and\ j}{Difference\ in\ infected\ averted\ instrategies\ i\ and\ j}$$

$$ICER(C) = \frac{0.4397}{63} = 0.0069793651$$

$$ICER(A) = \frac{0.2753 - 0.4397}{234 - 63} = -0.0009614035$$

The comparison between strategies C and A in Table 4 shows the cost of savings-0.0009614035 for

Strategy A over strategy C. The high ICER for strategy C indicates that strategy C is costlier and less effective than strategy A. Therefore, strategy C is excluded from the set of alternatives so it does not consume limited resources.

Table 5: Total infection averted, total cost and ICER.

Strategies	Total infections averted	Total costs $(\$)$	ICER
Strategy A	234	0.2753	0.023
Strategy D	288	0.4634	0.003483

The ICER is calculated as follows;

$$ICER(A) = \frac{0.2753}{12.0399} = 0.023$$

$$ICER(D) = \frac{0.4634 - 0.2753}{288 - 234} = 0.003483$$

From Table 5, we compare the strategy A and D, the results shows a cost of savings 0.003483 for Strategy D over strategy A, Similarly, the high ICER for strategy A indicates that strategy A is costlier and less effective than strategy D. Therefore, strategy A is excluded from the set of alternatives so it does not consume limited resources.

Table 6: Total infection averted, total cost and ICER.

Strategies	Total infections averted	Total costs $(\$)$	ICER
Strategy D	288	0.4634	0.0016
Strategy B	289	0.2159	-0.2475

The ICER is calculated as follows;

$$ICER(D) = \frac{0.4634}{288} = 0.0016090278$$

$$ICER(B) = \frac{0.2159 - 0.4634}{289 - 288} = -0.2475$$

The comparison between strategies D and B in Table 6 shows a cost savings of -0.2475 for strategy B over strategy D. The lower ICER for Strategy B indicates that Strategy B is strongly dominated. That is, Strategy D is costlier and less effective than Strategy B. Strategy D has to be excluded from the set of alternatives since it consumes limited resources. Fromthe results presented, we conclude that strategy B is the control with chicken vaccination and treatment has the least ICER and therefore is the more cost-effective strategy.

4. CONCLUSION

In this paper, we have developed a deterministic model for the transmission of the ICD and three control strategies have been investigated. In this process, we have designed an optimal control problem that minimizes the cost for implementation of the controls of the disease while also minimizing the total number of infected chickens. First, we have demonstrated that optimal control exists and that it can be applied in terms of the solution to the optimality system. The Pontryagin's maximum principle has been used in deriving and analyzing the conditions for optimal control of the ICD with control strategies such as vaccination u_1, environmental cleanliness u_2 and treatment of infected chickens u_3 that minimize the spread of the disease. The numerical analysis shows that each strategy has the potential to control the transmission of the disease. However, numerical results show that susceptible chicken increases while the infected chickens decrease whenever the control is applied. The findings from the optimal control suggest that the disease may be eradicated from farmers by using vaccination and treatment control which have been found to be the most cost-effective. However, each control strategy presented in this paper definitely reduces the number of infected chickens. Therefore, we advise the community to use vaccination and treatment control which are the cost-effective optimal control strategies and are sufficient to combat the epidemic of ICD with limited resources.

Conflict of Interest

Authors declare that there is no conflict of interest

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